

Problem 2.8

[Difficulty: 2]

2.8 A velocity field is given by $\vec{V} = ax^3\hat{i} + bxy^3\hat{j}$, where $a = 1 \text{ m}^{-2}\text{s}^{-1}$ and $b = 1 \text{ m}^{-3}\text{s}^{-1}$. Find the equation of the streamlines. Plot several streamlines in the first quadrant.

Given: Velocity field

Find: Equation for streamlines; Plot streamlines

Solution:

Streamlines are given by
$$\frac{v}{u} = \frac{dy}{dx} = \frac{b \cdot x \cdot y^3}{a \cdot x^3}$$

So, separating variables
$$\frac{dy}{y^3} = \frac{b \cdot dx}{a \cdot x^2}$$

Integrating
$$-\frac{1}{2 \cdot y^2} = \frac{b}{a} \cdot \left(-\frac{1}{x}\right) + C$$

The solution is
$$y = \frac{1}{\sqrt{2 \cdot \left(\frac{b}{a \cdot x} + C\right)}}$$

Note: For convenience the sign of C is changed.

$a = 1$

$b = 1$

$C = 0 \quad 2 \quad 4 \quad 6$

x	y	y	y	y
0.05	0.16	0.15	0.14	0.14
0.10	0.22	0.20	0.19	0.18
0.20	0.32	0.27	0.24	0.21
0.30	0.39	0.31	0.26	0.23
0.40	0.45	0.33	0.28	0.24
0.50	0.50	0.35	0.29	0.25
0.60	0.55	0.37	0.30	0.26
0.70	0.59	0.38	0.30	0.26
0.80	0.63	0.39	0.31	0.26
0.90	0.67	0.40	0.31	0.27
1.00	0.71	0.41	0.32	0.27
1.10	0.74	0.41	0.32	0.27
1.20	0.77	0.42	0.32	0.27
1.30	0.81	0.42	0.32	0.27
1.40	0.84	0.43	0.33	0.27
1.50	0.87	0.43	0.33	0.27
1.60	0.89	0.44	0.33	0.27
1.70	0.92	0.44	0.33	0.28
1.80	0.95	0.44	0.33	0.28
1.90	0.97	0.44	0.33	0.28
2.00	1.00	0.45	0.33	0.28

